

# Access charge and imperfect competition\*

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November 2002

## Abstract

A benevolent social planner, which faces a cost of public funds because of distortive taxation, wants to finance an upstream monopoly. This monopoly produces a necessary input for a downstream competitive sector which competes *à la Cournot* (with either a fixed number of firms or free entry in the downstream sector). We show that in both cases an ad valorem access charge is a better regulatory tool than a per unit access charge if the access charges are restrained to be positive. The reverse holds when access charges are used to subsidize the downstream market. We then analyze the incidence of the imperfect competition on final prices.

JEL codes: H22, L13, L51

Keywords: Ad valorem tax, Essential facility, Regulation, Specific tax, Vertical relations

This work is forthcoming in *Louvain Economic Review*.

## 1 Introduction

In the past few years, significant network economic sectors have been deregulated, such as telecommunications, postal services, air or rail transportation. Most of the time, monopolistic situation was the main concern of these deregulations which induced deep changes in the monopolies' economic environment and activities. Broadly speaking, some activities previously dedicated to a monopoly are now open to competition, while some others remain exclusively to the monopoly. It is often the case that this monopoly constitutes an essential facility for the production of the newly liberalized activities. In that sense, the remaining monopolistic sector is the upstream one and the liberalized one the downstream sector.

An example can be found in the telecommunication industry. Until recently, the local telecommunications have been, in France and in other countries, under the responsibility of the former

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\*We thank Helmuth Cremer and Jacques Crémer and an anonymous referee of the *Louvain Economic Review* for their help and comments. We also thank John Turtle and the participants at the European Economic Association congress (EEA 2000, Bolzano), at the European Association for Research in Industrial Economics conference (EARIE 2000, Lausanne), at the Journées de Microéconomie Appliquée (JMA 1999, Lyon) and at the Enter Jamboree (London) for their useful remarks, especially Carmelo Rodriguez.

telecommunication monopoly, while inter-regional telecommunications have been open to competitors a long time ago. Inter-regional calls need to be originated and terminated by local telecommunication loops, which belong to the monopoly. This point has been under debate in several competition authorities all around the world and the European Commission pushed in favor of the deregulation of the local telecommunication market. But this has not been achieved easily. In France for example, an Administrative Court in Nancy denied authorization to a group of towns to give private firms access to their optic fiber network, and to subcontract the management of this network to a private firm.<sup>1</sup>

Nevertheless, following the deregulation process, concrete attempts have been made by the regulators to authorize either the unbundling of the “copper” local loop (in order for competitors to be connected as near to the customers as possible and, thus, to allow them to provide more added value services) or alternative local loop technologies (such as TV cable or wireless local loop that allow competitors to completely by-pass the incumbent operator) and, the 1st of January 2002, each customer should be free to choose its operator in the local telephony market. Even if the necessary regulatory legal enforcement is often currently in place, it is, first, not yet of a significant importance in most of the European countries and, second and more importantly, all these solutions do not avoid the access charge problem. Unbundling, e.g., still requires access to the final customers to be authorized (and be priced) to other alternative operators by the remaining monopoly whenever by-pass has not been achieved.

Moreover, new competitive markets, which were not previously regulated, are raising where the monopoly faces competitors, but for which the monopoly (upstream) product constitutes also an essential facility. This is the case, for example, for mobile telephony<sup>2</sup> and for Internet access. Both products need, in different ways, the local telecommunication loop: in order to deliver some calls for the mobile telephone company, or in order to be accessed by consumers for Internet access providers.

All these changes do not avoid the need for regulation. To follow our telecommunication example, there is currently a virulent debate in the US and in Europe in order to set what status Internet access providers<sup>3</sup> should be provided: are they telecommunication companies – and thus should pay and receive access charges for incoming and outgoing calls, but also participate to the financing of universal service for telecommunication – or are they simple customers of telecommunication firms?

In fact, the scope of the regulation in the local and long-distance telecommunication sector moved from a control over the local and inter-regional prices, to a scrutiny of the local prices and of the interconnection prices among networks.<sup>4</sup> As shown by the preceding example, universal service also plays an important place in the regulatory agenda, as well as the appropriate structure of the market (should the upstream monopoly firm be allowed to operate in the downstream sector?) among other aspects. This paper focuses on the interconnection aspect.<sup>5</sup>

The interconnection price, or access charge, is the price paid by a downstream competitor to the upstream monopoly in order to access the upstream network. More precisely, an access charge is the price paid by any network which wants to access another network in order to provide the good or services sold to the end user. This price can take several forms. In the most common cases, it can be a constant per unit price, which will be referred to as “specific” access charge. It can also take

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<sup>1</sup>Source: Le Monde, 20th of March, 1999.

<sup>2</sup>Mobile telephony is also regulated. But the regulation is not of the same scope and strength than for the usual telephony through copper lines.

<sup>3</sup>Please refer to European Commission (01/10/1998) and Werbach (1997) for insights on this subject.

<sup>4</sup>These changes also raise the point of the appropriate structure of the market. The question about the opportunity to let the monopoly operate or not in the downstream sector is one example of the problems that emerge.

<sup>5</sup>For a good development related to universal service, please refer to Laffont and Tirole (2000, section 6) and Caillaud and Tirole (2000).

the form of a percentage of the revenues generated by the activity for which access in an essential facility. Such a charge will be denoted as “ad valorem”. Both names refer to the literature of public economics.

This paper focuses on one-way networks and does not consider the possibility that the access charge can be asked by a downstream firm to the upstream monopoly,<sup>6</sup> that is, following the analogy with the telecommunication sector, by a long distance operator to the local upstream monopolist network.<sup>7</sup> The question of the access charge has been recently the focus of numerous papers with, in particular, Laffont and Tirole (1994), Laffont and Tirole (2000), Armstrong, Doyle and Vickers (1996) or Armstrong (2001). In Laffont and Tirole (1994), for example, the upstream monopoly also competes in the downstream sector with a perfectly competitive fringe which produces an homogeneous good differentiated from the monopoly’s one. The final good is made of one good from the upstream monopoly plus one good produced either by the monopoly in the downstream sector or by the fringe. They study the optimal pricing under the assumption that the fringe pays an access charge based on the quantity of goods it produces, or specific access charge.

The main goal of this paper is to introduce imperfect competition in the fringe and to see what are the incidences of this imperfect competition on the regulatory policy and in particular on final prices. In their paper, Laffont and Tirole show that the optimal access charge is higher than the monopoly’s marginal cost to produce the upstream good. They interpret this mark-up as a tax that enables the social planner to raise funds in order to finance part of the monopoly’s fixed cost. But, as stated by the public economics literature, if there is no need to differentiate between an *ad valorem* or a specific taxes (or access charges) in a perfectly competitive market, the introduction of an imperfectly competitive fringe yields a necessary distinction of the two taxes because they become two regulatory tools with different effects. Thus, the introduction of imperfect competition has an incidence on the choice of access charge type.

The comparison between specific and *ad valorem* taxation is an old topic in public economics.<sup>8</sup> Recently, there has been several works on different models of imperfect competition in different contexts. In particular, Delipalla and Keen (1992) show that, with a Cournot oligopoly (with and without free entry), predominantly *ad valorem* taxation leads to relatively low price and low profits for the firms. Skeath and Trandel (1994) prove that, for some markets, *ad valorem* taxation Pareto dominates a specific one (higher consumer surplus, larger fiscal revenues and profits). In the context of international trade, Kowalczyk and Skeath (1994) show that *ad valorem* tariffs are better than specific tariffs in the case of a country importing from a foreign monopoly.

Thus, this paper aims, first, to point out the similarities and common features between public economics and network access pricing literatures and, as an application, to study what kind of regulatory tool, specific or *ad valorem*, is needed in an access pricing framework with downstream imperfect competition.

This paper is organized the following way. Section 2 describes the basic trade off between *ad valorem* and specific access taxes. Section 3 sets the framework of the economy under scrutiny and the conditions under which it is studied. Section 4 describes the behavior of the oligopoly when

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<sup>6</sup>Please refer to Armstrong (2001), Estache and Valetti (1999), Laffont and Tirole (1996) and Laffont and Tirole (2000) for more on the network aspects.

<sup>7</sup>Section 5 of Laffont and Tirole (2000) as well as section 4 of Armstrong (2001) are dedicated to the study of two-way access networks. Moreover, Armstrong (1998), Carter and Wright (1999), Dessein (1998), Laffont, Rey and Tirole (1998a), Laffont, Rey and Tirole (1998b) and Wright (2000) also deal with this same issues.

<sup>8</sup>The first intuitions about the differences between the two taxes are due to Cournot (1838) and Wicksell (1896). Afterwards, Suits and Musgrave (1955) showed that more tax revenue could be raised by an *ad valorem* taxation rather than by unit taxation in a monopoly case. Please refer to Myles (1995, chapter 11), Keen (1998) and Boldron (2001, chapter 3) for some insights on the subject.

facing taxes, or access charges, set by the regulator. In particular, the marginal effects of each tax on some of the fundamentals of the economy are derived. Next, section 5 gives the programs to be solved by the regulator. Section 6 demonstrates the superiority of *ad valorem* access charge over a specific one when these charges are restricted to be positive. Section 7 exhibits the optimal final prices associated with a positive *ad valorem* access charge. Section 8 analyzes the case when taxes are not restricted at all. Finally, section 9 sums up the results, discusses further extensions and concludes. All the proofs are detailed in the appendix.

## 2 Tradeoff between *ad valorem* and specific taxes in public economics

There are two main differences between the effects induced in an economy by a specific or an *ad valorem* tax. The first effect is related to fiscal revenues. Let consider any firm with constant marginal cost  $c$ , producing  $q$  for a market with consumer price  $p$ . When this firm faces a specific tax  $t_s$ , its profit function is

$$\pi = p \cdot q - c \cdot q - t_s \cdot q.$$

In the presence of an *ad valorem* tax  $t_v$ , its profit function becomes

$$\begin{aligned} \pi &= (1 - t_v) p \cdot q - c \cdot q \\ &= (1 - t_v) \left[ p \cdot q - c \cdot q - \frac{t_v \cdot c}{1 - t_v} \cdot q \right]. \end{aligned}$$

Thus, an *ad valorem* tax is a combination of both a profit tax  $t_v$  and a unit tax  $t_v \cdot c / (1 - t_v)$ . Therefore, if the specific tax is such that  $t_s = \frac{t_v \cdot c}{1 - t_v}$ , both taxes impact the firm's behavior the same way because they induce the same marginal cost function, ending with identical final quantity  $q$ . Nevertheless, whenever final profits are strictly positive (with such an additional marginal cost), the *ad valorem* tax raises more fiscal revenues than the specific one.

The second effect is related to price. When a firm is facing both taxes, its perceived price is  $\tilde{p} = (1 - t_v) p - t_s = p - (t_s + t_v \cdot p)$ . Thus, the (negative) markup over market price is constant for the firm whenever there is only a specific tax. Nevertheless, in the presence of an *ad valorem* tax, this markup varies with firm's output/price decision.

The basic trade-off related to specific and *ad valorem* taxes are related to these two main points: an *ad valorem* tax raises more fiscal revenues than a specific tax for a given identical firm's output decision and price faced by the producer is variable under an *ad valorem* tax while constant with a specific one.

## 3 Framework

The framework considered is inspired by Laffont and Tirole (1994). An industry produces two goods for consumers. Good 0 is produced by the upstream sector and good 1 by the downstream sector, using good 0 as an input. The production of one unity of good 1 requires one unity of good 0. Moreover, good 0 is assumed to have no substitute for the production of good 1. In other words, good 0 is an essential facility for the production of good 1.

In all this paper, the word “tax” is a generic term for both access charge (a positive tax) and subsidy (a negative tax). The latter terms are often preferred when there is no possible misunderstanding. Moreover, access charges usually refer to money raised by and for the upstream monopolist in exchange for its product. Here, the access charges are collected by the regulator.

**Consumers.** Consumers have an aggregated surplus  $S_0(Q)$  and  $S_1(Q)$  when consuming the quantity  $Q$  of, respectively, good 0 and good 1. These functions are assumed to verify  $S_{0Q}, S_{1Q} > 0$  and  $S_{0QQ}, S_{1QQ} < 0$ . The inverse demand functions of good 0 and 1 are, respectively,  $p_0(Q)$  and  $p_1(Q)$ , with negative derivatives  $p_{0Q}(Q)$  and  $p_{1Q}(Q)$ . The elasticities of demand for goods 0 and 1 are  $\eta_0 = -\frac{p_0}{Q_0 p_{0Q}}$  and  $\eta_1 = -\frac{p_1}{Q_1 p_{1Q}}$ . All functions are assumed to be continuously differentiable.

**Upstream sector.** The upstream sector is assumed to be a monopoly, producing  $q_0$  for the market of the good 0 alone and  $Q_1$  for the downstream market, at a cost  $C_0(q_0 + Q_1)$ , which is assumed to be continuously differentiable. There is a fixed cost  $C_0(0) > 0$  and the marginal cost is noted  $C_{0q}(q)$ . This monopoly is regulated in the sense that the price of the good 0 is set by the regulator and that the monopoly does not interfere with the production choice of good 1.

**Downstream sector.** The downstream sector is composed of  $n$  firms indexed by  $i$ . Each firm produces the same homogeneous good in quantity  $q_{1:i}$  at a cost  $C_i(q_{1:i})$ , with  $C_i(0) = 0$  and a marginal cost  $C_{iq}(q_{1:i})$ , and gets a profit  $\pi_i$ . Both cost functions are assumed to be continuously differentiable. In order to produce  $q_{1:i}$  units of the final good, a downstream firm needs  $q_{1:i}$  units of the intermediate good produced by the upstream monopoly. The aggregated production is then equal to  $Q_1$  and the aggregated profit of the oligopoly is denoted  $\Pi_1 = [(1 - t_v)p_1(Q_1) - t_s]Q_1 - \sum_i C_i(q_{1:i})$ . For expositional simplicity, all firms  $i$  are assumed to have the same cost function  $C_1(q_1)$  and are treated as a representative firm indexed by 1.

Moreover, to give the model the widest possible interpretation, the conjectural framework is used, following the formalization indicated in Delipalla and Keen (1992).<sup>9</sup> The conjecture, common to all firms, of the variation of total output induced by a marginal change of firm  $i$ 's production is denoted by  $\alpha = \frac{dQ_1}{dq_{1:i}}$ , with  $0 < \alpha \leq n$ , and denote  $\gamma = \frac{\alpha}{n}$ . The case  $\alpha = 1$  (or  $\gamma = \frac{1}{n}$ ) corresponds to the Cournot equilibrium, the Bertrand's one is approached as  $\alpha$  tends to 0 (or  $\gamma = 0$ ), and the case  $\alpha = n$  (or  $\gamma = 1$ ) represents tacit collusion among all firms of the oligopoly. As all firms share identical cost functions and conjecture, only symmetric equilibria are studied.

**Regulator.** The regulator wants to maximize social welfare which is the sum of the surplus of the consumers and the profits of the monopoly and the oligopoly.<sup>10</sup> But the regulator has to take into account the shadow cost of the public funds associated with the money it raises in order to finance, for example, the monopoly. To be more precise,<sup>11</sup> when the regulator wants to transfer funds from consumers to the monopoly through the fiscal system, this transfer induces a social cost  $\lambda$ , i.e. 1

<sup>9</sup>Conjectural variations concept, which date back to Bowley (1924) and Frisch (1933), have been heavily criticized for, among other things, the fact that almost any final behavior can be generated by an appropriate choice of conjectures. Some refinements have been introduced that made the conjecture endogenous as, for example, in Boyer and Moreaux (1983) but difficulties still remain. Nevertheless, they still provide an easy tool to describe several situations at the same time. Please refer to Jean-Marie and Querou and Tidball (2000) for both a survey of the literature on conjectural variations equilibrium and their properties.

<sup>10</sup>Consumers' surplus is used as a measure of social welfare, which is correct under some assumptions such as the absence of income effects in their utility function. This is motivated by the difficulty to deal with both a general equilibrium analysis and imperfect competition. Please refer to Salanié (1998, chapter 7) and Vives (1999, chapter 3).

<sup>11</sup>Please refer to Laffont and Tirole (1993) for a detailed analysis of this framework.

euro given to the firm has a cost in term of social welfare of  $(1 + \lambda)$  euros where  $\lambda$  euro is a pure economic loss. The underlying assumption driving this shortcut is the inability of the regulator to use lump sum transfers to collect money, which is replaced by a distorting taxation system. This is a second best setting.<sup>12</sup>

In this model, the regulator has to determine whether or not it has to intervene in the downstream market and, if so, what kind of access charge to demand. In other words, it has to choose the optimal level of taxes (no intervention corresponds to taxes equal to zero). Without loss of generality, the regulator is assumed to reimburse the costs of the monopoly dedicated to the upstream sector and, on top of that, it gives to the monopoly a monetary transfer  $T$ . Moreover, the regulator is credited with the payments induced by the sell of the upstream good  $q_0$  and with the revenues from the access charges.

In order to maximize social welfare, the regulator controls the price  $p_0$  of the upstream good, the monetary transfer  $T$ , the specific access charge  $t_s$  and the *ad valorem* one  $t_v$ . Of course, the analysis can be extended with other forms of access charge, like digressive sales taxes,<sup>13</sup> but only the most common commodity taxes are considered here.

The regulator is assumed to observe the cost of the upstream monopoly. The observability of the cost of the upstream monopoly is an important drawback from the Laffont and Tirole (1994) setting, where the distortion is created by the asymmetry of information. Our aim is to concentrate on the taxation distortion. Therefore, this informational distortion is not taken into account, i.e. it is assumed that there exists no asymmetric information between the regulator and the monopoly.

The constraints of the regulator are the following: the output and the oligopolist profits have to be positive (and the profit null in the free entry case), the output and the access charges have to be connected to the behavior of the oligopoly and the consumers. For the moment, there is no restriction on the taxes except that the *ad valorem* should not be greater than one ( $t_v \leq 1$ ).

**Economic structures of the industry.** As in the paper of Delipalla and Keen (1992), two scenario are considered: in the first one,  $n$  is fixed exogenously – case hereafter called Generalized Cournot; in the second one,  $n$  is determined endogenously with the zero profit condition – case hereafter called free entry oligopoly. In the latter case,  $n$  is treated as a real number.

**More notations.** A few more notations are needed in order to simplify the computations. Five aggregates are defined

$$\left\{ \begin{array}{l} K_1 = -\frac{C_{1qq}(q_1)}{\alpha(1-t_v)p_{1Q}(Q_1)}, \text{ with } \text{sgn}(K_1) = \text{sgn}(C_{1qq}) \\ K_2 = -\frac{p_{1QQ}(Q_1)q_1}{p_{1Q}(Q_1)}, \text{ with } \text{sgn}(K_2) = \text{sgn}(p_{1QQ}) \\ K_3 = \frac{C_{1q}(q_1)+t_s}{1-t_v} > 0 \\ K_4 = \frac{p_1(Q_1)(1+K_1)+K_3}{2+K_1} \\ K_5 = C_1(q_1) - q_1C_{1q}(q_1) \end{array} \right.$$

Aggregate  $K_3$  is the perceived marginal cost of the oligopoly when it faces the two taxes (recall section 2). Aggregate  $K_5$  measures the influence of the cost structure of the downstream sector.

<sup>12</sup>The introduction of imperfect downstream competition is another distortion on top of the cost of public funds. The absence of control over the competitive firms leads, in fact, to a third best world. When there is no shadow cost of public fund, i.e.  $\lambda = 0$ , we move back in a first best setting and the optimal policy of the regulator is to ensure a marginal cost pricing and to use lump sum transfers to restore positive profits (in fact zero profit) for the firms.

<sup>13</sup>Recently, Hamilton (1999) studied oligopoly taxation in a more general way.



It is positive whenever mean cost of production is higher than marginal cost. This occurs with a concave cost function (recalling that  $C_i(0) = 0$ ), that is for production technologies characterized by decreasing marginal cost (with or without fixed cost), by strictly positive fixed cost and constant marginal cost, or even by strictly positive fixed cost and (not too much) increasing marginal cost (as far the cost function is concave on the whole production range). Aggregate  $K_1$  and  $K_2$  show up in second order conditions and it is assumed that  $2 + K_1 > 0$ . This secures  $p_1 > K_4 > 0$ . Aggregate  $K_4$  is a parameter influencing the marginal effect of taxes.

Let us now turn to the analysis of the behavior of the oligopoly and derive comparative statics which will be helpful for the comparison of the two instruments.

## 4 Symmetric oligopoly with homogeneous good

### 4.1 Symmetric Generalized Cournot

**Behavior of the oligopoly.** In the framework described in section 3, the profit of a representative firm 1 of the downstream oligopoly is given by

$$\begin{aligned}\pi_1 &= [(1 - t_v) p_1(Q_1) - t_s] q_1 - C_1(q_1) \\ &= (1 - t_v) \left[ p_1(Q_1) q_1 - \frac{t_s q_1 + C_1(q_1)}{1 - t_v} \right].\end{aligned}\quad (1)$$

Thus, the representative firm's perceived marginal cost is  $K_3$ . When facing both taxes, the representative firm chooses its output level of good:  $q_1(t_s, t_v, n, \alpha)$ . This choice is driven by the first and second order conditions of the profit maximization on  $q_1$ , i.e. (without the arguments)

$$\frac{d\pi_1}{dq_1} = (1 - t_v) [\gamma p_1 Q_1 + p_1 - K_3] = 0, \quad (2)$$

$$\frac{d^2\pi_1}{dq_1^2} = \alpha(1 - t_v) p_1 Q_1 [2 + K_1 - \alpha K_2] < 0, \quad (3)$$

where  $\gamma = \frac{\alpha}{n}$ .

One implication of the first order condition is that  $p_1 > K_3$ , which means that the price on the downstream market is greater than the perceived marginal cost. This must be the case as firms enjoy some market power in this market: the higher  $\gamma$ , that is the higher the coordination among firms, the higher the mark-up over the perceived marginal cost. This yields the following implicit function

$$\frac{p_1 - C_1 q}{p_1} = t_v + \frac{t_s}{p_1} + \frac{\gamma(1 - t_v)}{\eta_1}, \quad (4)$$

$$\text{i.e., } t_v = 1 - \frac{C_1 q + t_s}{p_1 + \gamma p_1 Q_1} \quad (5)$$

and, from equation (3), the condition that  $2 + K_1 - \alpha K_2 > 0$ . A stronger condition is imposed by assuming the stability condition of Seade (1980):  $1 + \gamma(1 + K_1 - nK_2) > 0$ . Equation (4) exhibits where the mark-up over the marginal cost of production comes from: first, *ad valorem* and specific taxes (the first two terms), when positive, increase the price for consumers and, second, the effect of market power (third term) cannot be counterbalanced unless demand is perfectly elastic or the *ad valorem* tax leaves zero profit to the firms.

**Marginal effect of taxes.** Applying the implicit function theorem to equation (2) gives the effects of taxes on good 1's price. The effects on the oligopolistic profits need the differentiation of equation (1), where the first order condition (2) is used in order to simplify the computations. Thus, in a symmetric Generalized Cournot framework, taxes affect the oligopolistic behavior the following way

$$\frac{dp_1}{dt_s} = \frac{1}{(1-t_v)[1+\gamma(1+K_1-nK_2)]} > 0, \quad (6)$$

$$\frac{dp_1}{dt_v} = K_3 \frac{dp_1}{dt_s} > 0, \quad (7)$$

$$\frac{d\Pi_1}{dt_s} = \left[ (1-t_v)(1-\gamma) \frac{dp_1}{dt_s} - 1 \right] Q_1, \quad (8)$$

$$\frac{d\Pi_1}{dt_v} = \left[ (1-t_v)(1-\gamma) \frac{dp_1}{dt_v} - p_1 \right] Q_1 < p_1 \frac{d\Pi_1}{dt_s}. \quad (9)$$

An increase in any of the two taxes increases the final price and decreases the aggregated and individual quantities. But the effect on aggregated profits  $\Pi_1$  is ambiguous. Even when the economic background is such that an increase in any tax increases the total pie  $p_1 Q_1 - nC_1$ , one can have that fiscal revenues and profits increase, or that only one of them increases. This depends on the market structure, the level of taxation, and that is why the effect of taxes on profits is ambiguous. For instance, suppose that there is no over-shifting of specific taxation, i.e.  $dp_1/dt_s < 1$ . From equations (8) and (9), this is a sufficient condition to have decreasing profits with the two types of tax.

Moreover, one can remark that taxes affect price and profit the same way as in Delipalla and Keen (1992).<sup>14</sup> As they noted, broadly speaking, more *ad valorem* taxation leads to relatively low price and low profit. From an empirical point of view, there are not much work comparing specific and *ad valorem* taxation but they all tend to confirm that specific taxes lead to higher price.<sup>15</sup>

**A note on the effect of the number of firms.** When the number of firms increases, the competition between those firms is tougher and the equilibrium price tends to be lower.<sup>16</sup>

**Lemma 1.** *In the Generalized Cournot framework with constant or increasing marginal cost in the downstream sector, an increase in the (exogenous) number of firms increases the ad valorem tax that keeps the prices of goods 0 and 1 constant (in the context where the specific tax is equal to zero).*

On the one hand, more firms generate more fiscal revenues for the regulator (as the price of good 1 is constant) and increase the social welfare. But, on the other hand, the social welfare decreases because of the fixed costs replication. Thus, there is a strong incentive for the regulator to control the number of active firms on the market.

<sup>14</sup>In their article, Delipalla and Keen (1992) discuss the case of tax over-shifting.

<sup>15</sup>In his paper, Keen (1998) quotes three studies: Barzel (1976), Johnson (1978) and Delipalla and O'Donnell (1998).

<sup>16</sup>Proofs are in the appendix.



## 4.2 Symmetric free entry oligopoly

**Behavior of the oligopoly.** In this framework, firms have zero profit at the equilibrium, i.e.

$$\begin{aligned}
 0 &= [(1 - t_v) p_1 - t_s] Q_1 - n C_1, \\
 \text{or } t_v &= 1 - \frac{t_s Q_1 + n C_1}{p_1 Q_1}, \\
 \text{or } t_s &= (1 - t_v) p_1 - \frac{n C_1}{Q_1}.
 \end{aligned} \tag{10}$$

In the free entry oligopoly, the cost structure is constrained by  $K_5 > 0$ . Indeed, combining the zero profit condition (10) with the first order condition (2) yields  $K_5 = -(1 - t_v) \alpha (q_1)^2 p_1 Q_1 (Q_1) > 0$ .

The downstream individual output  $q_1(t_s, t_v, \alpha)$  and the number of active firms  $n(t_s, t_v, \alpha)$  are the solutions<sup>17</sup> of the first order condition (2), the second order inequality (3) and the zero profit condition (10).

**Marginal effect of taxes.** Using the implicit function theorem to equations (2) and (10) and the simplifications that, from equation (2),  $K_3 = \gamma p_1 Q_1 + p_1$ , and that, from equation (10),  $p_1 Q_1 = \frac{t_s Q_1 - n C_1}{(1 - t_v)}$ , one can find the following system

$$\begin{bmatrix} 1 + \gamma(1 + K_1 - n K_2) & -\gamma q_1 p_1 Q_1 (1 + K_1) \\ q_1 + \frac{p_1 - K_3}{n p_1 Q_1} & -\frac{K_5}{n(1 - t_v)} \end{bmatrix} \begin{bmatrix} dp_1 \\ dn \end{bmatrix} = \frac{1}{1 - t_v} \begin{bmatrix} 1 & K_3 \\ q_1 & q_1 p_1 Q_1 \end{bmatrix} \begin{bmatrix} dt_s \\ dt_v \end{bmatrix}.$$

Using the same simplifications, the determinant of the matrix on the left can be written  $-\frac{K_5(2 + K_1 - \alpha K_2)}{n(1 - t_v)}$ , which is strictly negative because  $K_5 > 0$  in a free entry framework and the second order condition requires  $2 + K_1 - \alpha K_2 > 0$ . Thus, by inverting of this matrix, the taxes affect the oligopolistic behavior in a symmetric free entry oligopoly the following way

$$\begin{aligned}
 \frac{dp_1}{dt_s} &= \frac{2 + K_1}{(1 - t_v)(2 + K_1 - \alpha K_2)} > 0, \\
 \frac{dp_1}{dt_v} &= K_4 \frac{dp_1}{dt_s} > 0,
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 \frac{dn}{dt_s} &= -\frac{\alpha q_1 (2 + K_1 - n K_2)}{K_5 (2 + K_1 - \alpha K_2)}, \\
 \frac{dn}{dt_v} &= p_1 \frac{dn}{dt_s} + \frac{n(1 - \gamma)}{(1 - t_v)(2 + K_1 - \alpha K_2)}.
 \end{aligned}$$

Once again, an increase in any of the two taxes increases the final price and decreases the aggregated and individual quantities. Moreover, the effect on the number of firms in the downstream market is ambiguous, for the same reasons as in the Generalized Cournot framework.

Now that the oligopolistic response to the tax instruments is better known, let turn to the problem of the regulator.

<sup>17</sup>There is nothing that secures the uniqueness of  $n$ . If all entry costs are sunk cost, this would induce one solution. Please refer to Vickers (1989) for a detailed treatment of the subject.

## 5 The programs of the regulator

As stated in section 3, the regulator maximizes, over the transfer, the two taxes and the price of the good 0, social welfare which is written as the sum of the consumers surplus, the firms' profits and the cost of the funds raised by the regulator, or the benefit if the regulator gets more money than it gives, i.e.

$$\begin{aligned} SW = & [S_0(q_0) + S_1(Q_1) - p_0(q_0)q_0 - p_1(Q_1)Q_1] \\ & + [T] + [(1 - t_v)p_1(Q_1) - t_s]Q_1 - nC_1(q_1) \\ & - (1 + \lambda)[T + C_0(q_0 + Q_1) - p_0(q_0)q_0 - [t_s + t_v p_1(Q_1)]Q_1]. \end{aligned} \quad (12)$$

The constraints that the regulator faces are the following: the symmetry of the oligopoly, the oligopolistic behavior in reaction to the setting of the taxes, the participation condition of the oligopoly, the consumer behavior in reaction to the prices and the constraints on the taxes.

**Symmetric Generalized Cournot (GC).** The program of the regulator is to maximize, over  $T$ ,  $p_0$ ,  $t_s$  and  $t_v$ ,

$$\begin{aligned} SW_{GC} = & S_0(q_0) + S_1(Q_1) + \lambda[p_0(q_0)q_0 + p_1(Q_1)Q_1] \\ & - \lambda T - \lambda \Pi_1(q_1) - (1 + \lambda)[C_0(q_0 + Q_1) + nC_1(q_1)] \end{aligned} \quad (13)$$

subject to the constraints

$$\left\{ \begin{array}{ll} \text{Symmetric oligopoly:} & Q_1 = nq_1, \\ \text{Oligopoly conduct:} & (1 - t_v)[\gamma p_1 Q_1 + p_1] - t_s - C_{1q} = 0, \\ & [2 + K_1 - \alpha K_2] > 0, \\ \text{Oligopoly participation:} & [(1 - t_v)p_1 - t_s]Q_1 - nC_1 \geq 0, \\ \text{Monopoly participation:} & T \geq 0, \\ \text{Ad valorem tax:} & t_v \leq 1, \\ \text{Consumer:} & S_{0Q} = p_0 \text{ and } S_{1Q} = p_1. \end{array} \right.$$

**Symmetric free entry oligopoly (FE).** The program of the regulator becomes the maximization, over  $T$ ,  $p_0$ ,  $t_s$  and  $t_v$ , of

$$\begin{aligned} SW_{FE} = & S_0(q_0) + S_1(Q_1) + \lambda p_0(q_0)q_0 + \lambda p_1(Q_1)Q_1 \\ & - \lambda T - (1 + \lambda)[C_0(q_0 + Q_1) + nC_1(q_1)] \end{aligned} \quad (14)$$

subject to the constraints

$$\left\{ \begin{array}{ll} \text{Symmetric oligopoly:} & Q_1 = nq_1, \\ \text{Oligopoly conduct:} & (1 - t_v)[\gamma p_1 Q_1 + p_1] - t_s - C_{1q} = 0, \\ & [2 + K_1 - \alpha K_2] > 0, \\ \text{Free entry condition:} & [(1 - t_v)p_1 - t_s]Q_1 - nC_1 = 0, \\ \text{Oligopoly participation:} & n \geq 1, \\ \text{Monopoly participation:} & T \geq 0, \\ \text{Ad valorem tax:} & t_v \leq 1, \\ \text{Consumer:} & S_{0Q} = p_0 \text{ and } S_{1Q} = p_1. \end{array} \right.$$

**The optimization problem.** First, one can note that it clearly appears from equations (13) and (14) that the transfer  $T$  from the regulator to the upstream monopoly is always socially costly and therefore is optimally set to 0 in both cases. This is a standard result in the absence of asymmetric information.

Second as noticed by Delipalla and Keen (1992, p. 361), “it may (in most case will) be the case that these optimization problems have no solution interior to the implicit requirement of the Generalized Cournot model that profits be non-negative or to the requirement of the model of free entry oligopoly that there be at least one active firm”. Therefore, two strategies arise. The first one is to impose explicit constraints while leaving unrestrained the tax instruments. This is the one used by Myles (1996). The second one is to ignore some of the constraints, while using limited instruments. This is the one used by Delipalla and Keen (1992) and this paper.

## 6 Pure *ad valorem* access charge

In this section, taxes are restrained to be positive:  $t_v \geq 0$  and  $t_s \geq 0$ . This restriction occurs when it is forbidden for governments to subsidize unregulated private firms. This is the case, for example, in the telecommunication industry where entrants in deregulated markets pay some access charges to get connected to final consumers. In the railway industry, transportation companies also pay transportation fees, an access charge to the physical network,<sup>18</sup> and, more generally, physical networks make intermediate users pay a positive amount for the access to the infrastructure. Therefore, in this section, the only monetary transfers that are allowed in the downstream sector are the ones from the firms to the government.

The goal of this section is to show, without making the computation of the optimal level of both access charges, that the regulator will only use the *ad valorem* tool. There are mainly two techniques in order to assess the superiority of one or another pair of taxes. The first one is to take two different pairs and to compute the difference in the associated social welfare levels. The second is to select a specific path of tax variation and to analyze what is the influence of a move along this path in terms of social welfare. This is the technique used here with changes in taxes that lead to the same prices for goods 0 and 1.

**Symmetric Generalized Cournot.** In order to have an easier comparison, the downstream firms’ positive profit constraint is ignored. If the optimal price leads to negative profits, the problem of the choice of the regulatory tool should be revised.<sup>19</sup>

**Proposition 1.** *In a symmetric Generalized Cournot framework, when taxes are restrained to be positive and the constraint of positive oligopolistic profit ignored, the optimal tax or access charge structure is a pure ad valorem one.*

The key-point of the proof is that a pure *ad valorem* tax that leads to an equilibrium price  $\tilde{p}$  yields a greater fiscal revenues than any combination of *ad valorem* and specific taxes that leads to the same

<sup>18</sup>Nevertheless, this industry does not perfectly fit with the framework described in this paper. Indeed, the main problem for the regulator is the diversity of goods to be allocated, and the coordination among different transportation firms. Hariton (2002, chapter 3) studies the repeated allocation by an auctioneer of one good, let say an access right, when bidders are asymmetric with respect to the number of periods they require access. Bassanini and Pouyet (2000) studies the coordination problem that arises between infrastructure regulators in the presence of both domestic and international traffic. Please refer to Caillaud (2000) for a detailed discussion on issues raised in this industry.

<sup>19</sup>In that case, the regulator’s program is modified in the sense that the oligopoly binds its constraint of positive profit. The question is to compare the social welfare levels associated with, first, a mix of specific and *ad valorem* taxes, and, second, the highest *ad valorem* tax, both leading to zero oligopolistic profits.

price  $\tilde{p}$ . Equation (12) shows that the regulator prefers to raise 1 euro in fiscal revenues, which gives  $(1 + \lambda)$  euros, instead of 1 euro in profits, which only counts for 1 euro in the social welfare. As the regulator prefers fiscal revenues than profits, it will only use the *ad valorem* tool when taxes are restrained to be positive.

Moreover, the variation in social welfare along the path of taxes with constant prices for goods 0 and 1 is

$$dSW = -\lambda\gamma(Q_1)^2 p_{1Q} dt_v.$$

Under perfect competition ( $\gamma = 0$ ),  $dSW = 0$  and there is no gain to shift from the specific tax to the *ad valorem* tax. This is the standard result of the equivalence between *ad valorem* and specific taxes. Moreover, as one gets away from perfect competition ( $\gamma$  increasing), the marginal social value of the shift increases.

This result can be interpreted another way. If the regulator is constrained, for any reason, to use only one type of tax then, for a given final price, consumers prefer that the regulator uses *ad valorem* taxation whereas firms prefer a specific one. This is because, for a given final price, *ad valorem* taxation leads to higher fiscal revenues (and then less profits!). Of course, if consumers are not aware of the social cost of public funds, they are indifferent because final prices are the same. In that case, there is more room for capture of the regulator by the industry: “more room” because consumers are less able to discover the capture (same prices), but “capture” because firms are not indifferent in the kind of taxation used (they prefer specific).

**Symmetric free entry oligopoly.** With this type of imperfect competition, the profits of the downstream firms are always equal to zero but it is assumed that there is at least one active firm in the oligopoly.

**Proposition 2.** *In a symmetric free entry oligopoly framework, when taxes are restrained to be positive and the constraint that at least one firm is active ignored, the optimal tax or access charge structure is a pure ad valorem one.*

In the free entry framework, there is no profit for the downstream firms and the comparison of the two tools cannot be based on profit levels. Nevertheless, in this framework, the two taxes do not affect the number of active firm at equilibrium the same way. Take the case of constant marginal cost<sup>20</sup> and a positive fixed cost for the downstream sector. To prove the result, one can use the fact that with a pure *ad valorem* tax that leads to an equilibrium price  $\tilde{p}$ , there is less active firms than with any combination of *ad valorem* and specific taxes that leads to the same equilibrium price  $\tilde{p}$ . As the marginal cost is constant, the number of firms does not affect the total variable cost. Then, the decision of the regulator will be based on the number of active firm. The regulator prefers to have less active firms because it means less fixed cost and a higher social welfare, so the *ad valorem* tax is a better regulatory instrument.

**Reverse case: Pure specific subsidization.** Assume now that the reverse case occurs, i.e. the regulator can only use a mix of *ad valorem* and specific access charges which are restricted to be negative. This restriction could be an interesting case to study if, for instance, the regulator would like to subsidize an activity. Then, if it cannot use lump-sum transfers to finance Universal Service obligations but if it is still allowed to use access charge subsidization, the better regulatory tool is the specific access charge. In the telecommunication sector, this can occur were the regulator to subsidize, e.g., Internet access in areas with low density of population. In the railway industry, regional transportation through villages and small towns can also give rise to this concern.

<sup>20</sup>Of course, the proof holds for more general cost functions.

**Proposition 3.** *In a symmetric Generalized Cournot (respectively, free entry oligopoly) framework, when the taxes are restrained to be negative and the constraint of positive oligopolistic profit (resp., that at least one firm is active) is ignored, the optimal tax or subsidy structure is a pure specific one.*

Broadly speaking, the same reasoning applies as for propositions 1 and 2. For example, in the symmetric Generalized Cournot framework, the amount of money needed for a given price to finance the downstream market is greater with a combination of specific and *ad valorem* subsidies than with a pure specific subsidy. Therefore, achieving any given equilibrium price is less costly to obtain for the regulator with a specific subsidy.

## 7 Optimal prices

The goal of this section is to show the incidence of imperfect competition on the final price. Optimal prices of goods 0 and 1 are derived in the two proposed frameworks. In this section, the taxes are once again restrained to be positive.

In the two frameworks, the optimal prices correspond to the constrained maximization of the social welfare with respect to both taxes. Moreover, propositions 1 and 2 set that the optimal solution is characterized by  $t_s = 0$  in both frameworks. Therefore, the research of the optimal *ad valorem* access charge is simplified by the resolution of the equation for this particular value of the specific tax. This access charge fully determines the price of good 1, while the price of good 0 is directly controlled by the regulator. Therefore, the maximization can be done with respect to the access charge  $t_V$  or the price  $p_1$ .

**Benchmark.** In the case of perfect competition in the downstream sector, the optimal prices  $p_0^*$  and  $p_1^*$  for good 0 and 1 are such that<sup>21</sup>

$$\left\{ \begin{array}{l} \frac{p_0 - C_{0q}}{p_0} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_0} \\ \frac{p_1 - C_{0q} - C_{1q}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_1} \end{array} \right. \quad (15)$$

The prices are *à la Ramsey*, where  $\eta_i$  is the price elasticity of demand for good  $i$ .

**Symmetric Generalized Cournot.** The derivatives of social welfare, equation (13), with respect to  $p_0$  and  $p_1$  are

$$\left\{ \begin{array}{l} p_{0Q} SW_{p_0} = p_0 + \lambda q_0 p_{0Q} + \lambda p_0 - (1 + \lambda) C_{0q} (q_0 + Q_1) \\ p_{1Q} SW_{p_1} = p_1 + \lambda Q_1 p_{1Q} + \lambda p_1 - (1 + \lambda) [C_{0q} (q_0 + Q_1) + n C_{1q} (q_1)] - \lambda \Pi_{1p} \end{array} \right.$$

<sup>21</sup>These computations are quoted from the paper of Laffont and Tirole (1994, section 2) when there is complete information.

which give the two associated Lerner indexes.<sup>22</sup> Then, in a symmetric Generalized Cournot framework, when the taxes are restrained to be positive, the optimal prices are such that<sup>23</sup>

$$\left\{ \begin{array}{l} \frac{p_0 - C_{0q}}{p_0} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_0} \\ \frac{p_1 - C_{0q} - C_{1q}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_1} + \frac{\lambda}{1 + \lambda} \frac{p_{1Q}}{p_1} \Pi_{1p} \end{array} \right. \quad (16)$$

if the oligopolistic profit is positive, where  $\Pi_{1p} = \frac{d\Pi_1}{dp_1}$ .

In comparison to a price *à la Ramsey*, the optimal price of good 1 includes a corrective term linked to the profits made by the downstream firms. These profits are socially costly because they could have been used to finance the upstream monopoly in place of public funds. If the derivative of the oligopolistic profits at the optimum is positive, the corrective term is negative and the mark-up of the price to the marginal costs is decreased:<sup>24</sup> profits are socially costly and are limited through a price decrease. If the derivative is negative, the corrective term is positive and the mark-up increased: increasing the price reduces the profits and extracts enough fiscal revenues. It is difficult to be more predictive with regards to the sign of the derivative. Indeed, in this context,  $\Pi_{1p} = n \frac{d\pi_1}{dq} p_{1Q} + \frac{d\Pi_1}{dt_v} / \frac{p_1}{dt_v}$ . The first term is null and  $\Pi_{1p}$  is of the sign of  $\frac{d\Pi_1}{dt_v}$  which, as shown in section 4.1, can be either positive or negative. Thus, the sign of  $\Pi_{1p}$  is ambiguous.

The good 0 is still priced *à la Ramsey*. But, in general, imperfect competition has also an impact on its level, which may differ from the one with perfect competition. Indeed, the production of good 1 has an incidence on the level of the total quantity of good 0 produced and, therefore, it modifies the marginal cost of production of the upstream monopoly. As the quantities of good 1 produced are not the same, prices of good 0 will also be affected by imperfect competition. Of course, if the marginal cost of the upstream firm is constant, the price of good 0 is unaffected.

**Symmetric free entry oligopoly.** The derivatives of the social welfare, equation (14), with respect to  $p_0$  and  $p_1$  are

$$\left\{ \begin{array}{l} p_{0Q} SW_{p_0} = p_0 + \lambda q_0 p_{0Q} + \lambda p_0 - (1 + \lambda) C_{0q} (q_0 + Q_1) \\ p_{1Q} SW_{p_1} = p_1 + \lambda Q_1 p_{1Q} + \lambda p_1 - (1 + \lambda) \left[ C_{0q} (q_0 + Q_1) + \frac{d}{dQ_1} (nC_1(q_1)) \right] \end{array} \right.$$

with  $\frac{d}{dQ_1} (nC_1) = C_{1q} + (C_1 - q_1 C_{1q}) \frac{dn}{dQ_1}$ .

Therefore, in a symmetric free entry oligopoly framework, when the taxes are constrained to be positive, the optimal prices are such that<sup>25</sup>

$$\left\{ \begin{array}{l} \frac{p_0 - C_{0q}}{p_0} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_0} \\ \frac{p_1 - C_{0q} - C_{1q}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta_1} + \frac{K_5}{p_1} n_Q \end{array} \right.$$

<sup>22</sup>To ensure that  $t_v > 0$ , it is assumed that a marginal increase of  $t_v$  when  $t_v = 0$  has a positive effect on social welfare.

<sup>23</sup>One as to remember from section 5 that the monetary transfer  $T$  is equal to zero.

<sup>24</sup>Without constant marginal costs, the different mark-ups are not really of the same size, because the productions of good 1 are different in the two cases and influence the level of marginal cost  $C_{0q}$ . Despite that limit, the spirit of the reasoning remains true.

<sup>25</sup>Again, to ensure that  $t_v > 0$ , it is also assumed here that a marginal increase of  $t_v$  when  $t_v = 0$  has a positive effect on social welfare.



where  $n_Q = \frac{dn}{dQ_1}$ . The difference with a price *à la Ramsey* is the corrective term linked to the fact that free entry induces an endogenous replication of the fixed costs. This corrective term takes into account the incidence of the choice of  $p_1$  on the number of active firms at the equilibrium, which itself has an incidence on the social welfare level. The price of good 0 is still priced *à la Ramsey* but it will in general be different from the price of good 0 in case of perfect competition for the same reason as in the symmetric generalized Cournot framework.

## 8 Unrestricted regulatory tools

The aim of this section<sup>26</sup> is to modify the methodology used until now to study the access charge problem by leaving the tax instruments unrestricted and by focusing on the zero profit oligopolistic condition.

It is yet possible to have a first idea of why there may be an optimal mix of positive *ad valorem* and negative specific taxes. In section 18, the constraint on the specific tax was binding because of the constraint of positiveness. But decreasing the same specific tax along the specified tax path, i.e. leaving prices constant, would induce a better social welfare. Thus, moving from the situation where  $t_s = 0$  and  $t_v \leq 1$  to a situation where  $t_s < 0$  should increase social welfare. Nevertheless, it is rather difficult to find, in practice, instances of industries facing both negative and positive tax.

Recalling the benchmark exhibited in section 7, the regulator, when it imposes Ramsey prices  $p_0^*$  and  $p_1^*$  defined by equations (15), gets a revenue  $R^*$  such that

$$R^* = p_0^* q_0^* + p_1^* Q_1^* - C_0(q_0^* + Q_1^*) - nC_1\left(\frac{Q_1^*}{n}\right).$$

This is the revenue that gets the social planner from selling both goods 0 and 1 when it takes as given the number of firms producing good 1, ending up with a level of social welfare equal to

$$SW^* = S_0(q_0^*) + S_1(Q_1^*) - p_0(q_0^*)q_0^* - p_1(Q_1^*)Q_1^* + (1 + \lambda)R^*.$$

Another interpretation of  $R^*$  is that it represents the aggregate profits of private regulated firms operating in markets 0 and 1. In a private and unregulated environment, the government cannot face a better situation than when both markets are characterized by Ramsey prices. Define  $R(t_s, t_v)$  the function measuring the level of fiscal revenues collected by a pair of taxes  $(t_s, t_v)$

$$R(t_s, t_v) = p_0(q_0)q_0 - C_0(q_0 + Q_1) + [t_v p_1(Q_1) + t_s]Q_1.$$

The main goal of this section is to show that there exists a combination of specific and *ad valorem* taxes  $(t_s^*, t_v^*)$  that generates both Ramsey prices and fiscal revenues equal to  $R^*$ . If such a pair exists, then the profit of the downstream firms is zero

$$\begin{aligned} \Pi_1 &= [(1 - t_v^*)p_1^* - t_s^*]Q_1^* - nC_1^* \\ [R(t_s^*, t_v^*) = R^*] &= p_0^* q_0^* + p_1^* Q_1^* - C_0^* - nC_1^* - R^* \\ [\text{definition of } R^*] &= 0. \end{aligned}$$

<sup>26</sup>This section follows the paper of Myles (1996).

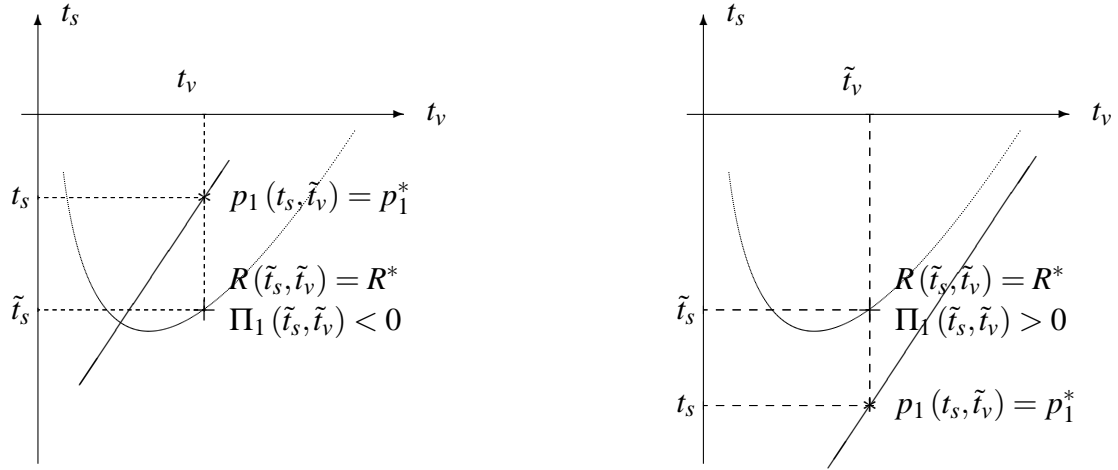


Figure 1: Relative positions of specific taxes yielding either fiscal revenues  $R^*$  or Ramsey price  $p_1^*$  and the impact in term of downstream profits

Moreover, when  $p_1^*$  is reached through some tax combination, then  $p_0$  is also set to its Ramsey level.<sup>27</sup> Therefore, one can focus, first, on good 1 and, second, on the existence of such a pair of taxes irrespective of the participation constraint of the downstream firms which is automatically satisfied for this particular pair. The following lemma describes the links between downstream profits and fiscal revenues generated by a pair of taxes  $(t_s, t_v)$ .

Two main assumptions are required to obtain this result. First, the Ramsey fiscal revenues  $R^*$  have to be small enough so that it can be generated by any of the two tax instrument. Second, all pairs of taxes that generate Ramsey price  $p_1^*$  have to be such that  $p_1 + p_{1Q}Q_1 < (1 - t_v)(p_1 + \gamma p_{1Q}Q_1) - t_s$ . Quoting Myles (1996, p. 36), this assumption “is equivalent to true marginal revenue being below perceived marginal revenue. Since the aim of the policy is to reduce the equilibrium price to the Ramsey level, this will be achieved by encouraging an expansion in output that is exactly what follows from raising perceived marginal revenue above actual marginal revenue”.

**Lemma 2.** *In the symmetric Generalized Cournot framework, if there exists a pair of taxes  $(\tilde{t}_s, \tilde{t}_v)$  such that, first,  $R(\tilde{t}_s, \tilde{t}_v) = R^*$  and, second, it does not generate the Ramsey price  $p_1^*$ , then the specific tax  $t_s$  that, associated with  $\tilde{t}_v$ , generates  $p_1^*$  is lower (respectively higher) than  $\tilde{t}_s$  if the oligopolistic profits generated with  $(\tilde{t}_s, \tilde{t}_v)$  are positive (resp. negative).*

Let define the *Ramsey fiscal revenues curve* as the curve, in a graph made of both taxes, made of all tax pairs that induce fiscal revenues equal to  $R^*$  and positive profits for the downstream oligopoly. Let also define the *Ramsey price curve* as the curve made of tax pairs that allows to generate  $p_1^*$ . As shown in section 4.1, an increase in any tax raises price  $p_1$  and the curve must be increasing. Then, lemma 2 asserts that the Ramsey price curve lies outside the Ramsey fiscal revenues curve whenever the firms earn positive profits on this later and, if there exists some pairs that induce negative profits for the oligopoly, then both curves must cross at least once by continuity. Figure 1 exhibits the two possible situations. The existence of this crossing point is insured by the first assumption on

<sup>27</sup>Equation (16) sets that the only way  $p_0$  can be modified from its Ramsey level is, if  $C_{0q}$  is not assumed to be constant, through a change in the quantity  $Q_1$  which in turn influences the marginal cost of producing good 0. But if the price of good 1 is set to  $p_1^*$  then the quantity of good 1 is equal to  $Q_1^*$  and the first order condition on  $p_0$  is equivalent to its first best counterpart.

the existence of both taxes level such that each of them can induce fiscal revenues greater than  $R^*$ . Moreover, reminding equations (8) and (9) from section 4, the profit is monotonically decreasing along the Ramsey price curve with the *ad valorem* tax

$$d\Pi_1|_{dp_1=0} = -(p_1 - K_3) Q_1 dt_v < 0$$

whenever  $dt_v > 0$ . This yields the following proposition.

**Proposition 4.** *In the symmetric Generalized Cournot oligopoly, if there exists a pair of taxes  $(\tilde{t}_s, \tilde{t}_v)$  such that  $\tilde{t}_v < 1$ ,  $R(\tilde{t}_s, \tilde{t}_v) = R^*$  and the oligopolistic profit generated by these taxes are negative, then there exists a unique pair  $(t_s^*, t_v^*)$  with  $t_v^* < 1$  that generates both fiscal revenues equal to  $R^*$  and Ramsey prices  $p_0^*$  and  $p_1^*$ .*

Under the two reasonable assumptions necessary to establish these results, proposition 4 demonstrates that Ramsey prices can be generated when firms become unprofitable at some combinations of taxes that satisfy a revenue requirement.<sup>28</sup> Therefore, the absence of restriction on taxes allow the regulator to recover a situation in which downstream firms have no market power. Of course, the possibility to have unrestricted taxes appears more socially beneficial than situations with restriction on taxes.

The intuition behind this result is quite simple. On the one hand, the *ad valorem* access charge limits the gradient of marginal revenue and, as a consequence, firms perceive their market power as reduced, which relax the upward pressure on prices. On the other hand, the specific access subsidy induces a reduction in the marginal cost of production. Both effects go in the same direction: price decrease. Thus, combining the two taxes allow to generate Ramsey prices while maintaining Ramsey fiscal revenues.

## 9 Conclusion

This paper shows that, depending on the economic background, the optimal regulatory tool for access charge is the *ad valorem* one (the optimal regulatory tool for subsidization being the specific one). Furthermore, the imperfect competition has an incidence on optimal price. Eventually, leaving access charges unrestricted allows to reach Ramsey prices, the best situation for a regulator in this framework, which eliminates welfare loss arising from downstream firms' market power. This is of particular interest for all industries where access is required to an homogenous essential facility and where market power in the newly deregulated markets are a serious concern. Regulators should, first, favor *ad valorem* pricing structures for these essential facilities instead of prices per unit in order to finance more efficiently the infrastructure. Second, they should try to play with both access charge instruments to limit market power. Nevertheless, it should be stresses that the exhibited dominance of *ad valorem* access charge would be moderated if the regulator has to take into account the quality of the final good. As Keen (1998) remarked, specific taxation leads to a relatively higher quality product.

There are many ways to extend this model. These extensions are related either to the taxation/access charge tools or to the structure of the industry. First, from the technical point of view, the case where the optimal *ad valorem* access charge leads to negative profits for the firm should

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<sup>28</sup>Myles (1996) develops furthermore its analysis. As he does not exhibit sufficient conditions for profits of the oligopoly to become negative at a point, Myles shows that even if the pair of taxes necessary to proposition 4 does not exist, the optimal policy is to let  $t_v$  reach the value 1 and to set  $t_s$  such that  $R^*$  is raised.

be analyzed with much scrutiny. It seems to be possible to find under which pertinent economic conditions the regulatory tools performs better one with respect to the other in this case.

Second, one should consider other usual tools to price access, such as profit taxation. Indeed, in a recent paper, Reinhorn (1999) argues that the tax choice should be affected by other type of fiscal instruments and in particular profit taxation. More generally, this stresses the question of non linear access charge/taxation. Nevertheless, when both taxes are unrestricted, profit taxation would not lead to any further gains in welfare. Indeed, in the imperfectly competitive framework used in this paper, distortions are created through pricing strategies. Thus, even if taxing profits raises funds without additional pricing distortions, there is no way it can correct current incentives for those distortions. Moreover, if both unrestricted taxes generate Ramsey fiscal revenues, then downstream firms make zero profit and profit taxation is a redundant tool.

Third, the *ad valorem* dominance (or specific in the case of subsidies) should be generalized to other forms of imperfect competition and in particular the study of price competition with imperfect substitute. This analysis should be based on the work of Anderson, de Palma and Kreider (2001) and Anderson, de Palma and Kreider (2001), who study particular forms of price competition. Moreover, with this kind of competition it should be easier to study the taxation of two-part tariffs which are common practice in several sectors, such as the telecommunication industry.

Turning to the extensions related to the structure studied in this paper, a specific emphasis should be given to the study of the tax choice incidence on the way to organize vertically related industries where natural monopolies supply an essential facility to potentially competitive sectors. In the context of liberalization of network industries, the basic questions one has to answer are, first, in which conditions the regulator should break the vertically integrated monopoly into one upstream regulated essential facility monopolistic producer and in a competitive downstream final good sector and, second, if the upstream monopoly should be authorized to compete in the downstream competitive market. These questions have been studied by, for example, Vickers (1995) and Lee and Hamilton (1998) but in a regulatory context where the contracts are based on a per unit access charge. Our intuition is that an *ad valorem* access charge could change the arbitrage exhibited in such works.

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## A Appendix

### A.1 Proof of lemma 1

For a given price  $p_1$  and  $t_s = 0$ , equation (5) defines the *ad valorem* tax as a function of the aggregated quantity  $Q_1$  and the number  $n$  of firms. Thus, the implicit derivation yields

$$\begin{aligned} \frac{\partial t_v}{\partial n} &= -\frac{\frac{\partial}{\partial n} [(1-t_v) (\frac{\alpha}{n} p_1 Q_1 + p_1) - (t_s + C_{1q})]}{\frac{\partial}{\partial t_v} [(1-t_v) (\frac{\alpha}{n} p_1 Q_1 + p_1) - (t_s + C_{1q})]} \\ &= \frac{Q_1 C_{1qq} - (1-t_v) \alpha p_1 Q_1}{n^2 \gamma p_1 Q_1 + p_1} \\ &= -\frac{\alpha p_1 Q_1 K_1 + 1}{(1-t_v) n^2 K_3}. \end{aligned}$$

The sign of the right-hand side expression is the sign of  $(1 + K_1)$ . Therefore, there is no general conclusion when marginal cost  $C_{1q}$  is decreasing because  $K_1$  is negative and can lie between  $-1$  and  $-2$  by the assumption that  $2 + K_1 > 0$ . Nevertheless, when marginal cost are increasing or constant,  $K_1$  is positive or null and therefore the derivative is positive.

### A.2 Proof of proposition 1

The technic used here is the one of Delipalla and Keen (1992, proposition 8). Let us take a pair of tax  $(t_s > 0, t_v > 0)$  such that it yields a positive profit for the oligopoly and respects the first-order condition

$$\begin{cases} 1 - t_v = \frac{(t_s + C_{1q})n}{np_1 + \alpha p_1 Q_1}, \\ [(1-t_v) p_1 - t_s] Q_1 - nC_1 \geq 0. \end{cases} \quad (10)$$

From this situation, consider the following shift from the specific tax to the *ad valorem* tax:  $K_3 dt_v = -dt_s > 0$ . Remembering that the marginal effect of taxes on  $p_1$  as described by (7) and that  $dp_1 = \left(\frac{dp_1}{dt_s}\right) dt_s + \left(\frac{dp_1}{dt_v}\right) dt_v$ , one gets  $dp_1 = 0$ . Therefore, with this particular shift, prices stay equal, as aggregated quantities. As  $n$  is fixed exogenously, individual quantities also stay equal.

Moreover, the transfer  $T$ , given by the regulator over the cost of production, is always costly for the social welfare and set to zero. Thus, using equation (13), the computation of the variation of the social welfare, at  $(t_s, t_v)$  and on this particular shift of taxes, can be written the following way

$$\begin{aligned} dSW &= \lambda Q_1 dt_s + \lambda p_1 Q_1 dt_v - d \left[ nC_1 \left( \frac{Q_1}{n} \right) \right] \\ [n \text{ exogenous}] &= \lambda Q_1 [dt_s + p_1 dt_v] \\ [K_3 dt_v = -dt_s] &= \lambda Q_1 [p_1 - K_3] dt_v \\ [\text{equation (2)}] &= -\lambda \gamma (Q_1)^2 p_1 Q_1 dt_v \\ dSW &> 0. \end{aligned}$$

This conclusion is true for every pair of taxes such that it sustains an oligopolistic equilibrium. But it does not insure that the new pair of taxes yields a positive profit for the oligopoly. Therefore, if the constraint of positive oligopolistic profit is ignored, the shift of taxes can be done until the constraint on  $t_s$  is reached, i.e.  $t_s = 0$  and the optimal pair of taxes is just made of an *ad valorem* tax.

### A.3 Proof of proposition 2

Let us take a pair of tax  $(t_s > 0, t_v > 0)$  such that it yields no profit for the oligopoly and respects the first-order condition

$$\begin{cases} 1 - t_v = \frac{t_s Q_1 + n C_1}{p_1 Q_{1:n}}, & (1) \\ 1 - t_v = \frac{(t_s + C_{1q})^n}{n p_1 + \alpha p_1 Q_{1:n}}. & (5) \end{cases}$$

From this situation, consider the following shift from the specific tax to the *ad valorem* tax:  $K_4 dt_v = -dt_s > 0$ . Remembering that the marginal effect of taxes on  $p_1$  as described by (11) and that  $dp_1 = \left(\frac{dp_1}{dt_s}\right) dt_s + \left(\frac{dp_1}{dt_v}\right) dt_v$ , one gets  $dp_1 = 0$ . Therefore, with this particular shift, prices stay equal, as aggregated quantities.

Moreover, the transfer  $T$ , given by the regulator over the cost of production, is always costly for the social welfare and set to zero. Thus, the computation of the variation of the social welfare on a move of taxes can be written the following way

$$\begin{aligned} dSW &= \lambda Q_1 dt_s + \lambda p_1 Q_1 dt_v - d \left[ n C_1 \left( \frac{Q_1}{n} \right) \right] \\ &\stackrel{[\text{equation (10)}]}{=} \lambda Q_1 [dt_s + p_1 dt_v] - d \left[ [(1 - t_v) p_1 - t_s] Q_1 \right] \\ &= (1 + \lambda) Q_1 [dt_s + p_1 dt_v] \\ &\stackrel{[K_4 dt_v = -dt_s]}{=} (1 + \lambda) Q_1 [p_1 - K_4] dt_v \\ dSW &> 0 \end{aligned}$$

because, as far as  $2 + K_1 > 0$ ,  $p_1 > K_4$ . As the conclusion is true for every pair of taxes such that it sustains an oligopolistic equilibrium, the shift of taxes can be done until the constraint on  $t_s$  is reached, i.e.  $t_s = 0$  and the optimal pair of taxes is just composed of an *ad valorem* tax. The dual problem of the Generalized framework is that this new pair of taxes does not insure that  $n \geq 1$ .

### A.4 Proof of proposition 3

**Symmetric Generalized Cournot framework.** The proof is as for the proposition 1. Let us take a pair of tax  $(t_s < 0, t_v < 0)$  such that it yields a positive profit for the oligopoly and respects the first-order condition. From this situation, consider the following shift from the specific tax to the *ad valorem* tax:  $K_3 dt_v = -dt_s > 0$ , i.e. the specific tax becomes more negative, while the *ad valorem* increases, but remains negative. This particular shift yields no change in prices, as in aggregated and individual quantities. The change in social welfare is given by  $dSW = \lambda Q_1 [p_1 - K_3] dt_v > 0$ . This conclusion is true for every pair of taxes such that it sustains an oligopolistic equilibrium. But it does not insure that the new pair of taxes yields a positive profit for the oligopoly. Therefore, if the constraint of positive oligopolistic profit is ignored, the shift of taxes can be done until the constraint on  $t_v$  is reached, i.e.  $t_v = 0$  and the optimal pair of taxes is just made of a specific tax.

**Symmetric free entry oligopoly framework.** The proof is as for the proposition 2. Let us take a pair of tax  $(t_s < 0, t_v < 0)$  such that it yields no profit for the oligopoly and respects the first-order condition. From this situation, consider the following shift from the specific tax to the *ad valorem* tax:  $K_4 dt_v = -dt_s > 0$ , i.e. the specific tax becomes more negative, while the *ad valorem* increases, but remains negative. This particular shift yields no change in prices and aggregated quantities. The change in social welfare is given by  $dSW = (1 + \lambda) Q_1 [p_1 - K_4] dt_v > 0$ . This conclusion is true

for every pair of taxes such that it sustains an oligopolistic equilibrium. But it does not insure that the new pair of taxes yields at least one active firm in the oligopoly. Therefore, if this constraint is ignored, the shift of taxes can be done until the constraint on  $t_v$  is reached, i.e.  $t_v = 0$  and the optimal pair of taxes is just made of a specific tax.

## A.5 Proof of lemma 2

Two assumptions are needed to demonstrate lemma 2 and proposition 4

$$\begin{cases} \exists (t_s^0, t_v^0) \text{ such that } R(t_s^0, 0) > R^* \text{ and } R(0, t_v^*) > R^* \\ \forall (t_s, t_v) \text{ inducing Ramsey prices, } p_1 + p_{1Q}Q_1 < (1 - t_v)(p_1 + \gamma p_{1Q}Q_1) - t_s \end{cases}$$

These two conditions are discussed in the main text. The proof follows the one of Myles (1996, lemma 1). Let us define

$$\Delta R(t_s, t_v) = p_0(q_0)q_0 + p_1(Q_1)Q_1 - C_0(q_0 + Q_1) - nC_1(q_1) - R^*$$

and remember that whenever a pair of taxes induces Ramsey price for good 1, it also induces Ramsey price for good 0. Thus, if a pair of taxes induces  $p_1^*$ , it is such that  $\Delta R = 0$ .

The derivative of  $\Delta R(t_s, t_v)$  with respect to  $t_s$  has first to be positive. As  $R^*$  is independent of  $(t_s, t_v)$ , one gets

$$\frac{\partial \Delta R}{\partial t_s} = [p_{1Q}Q_1 + p_1 - C_{0q} - C_{1q}] Q_{1t_s} > 0.$$

The sign comes from equation (6) with  $Q_{1t_s} < 0$  and from one assumption  $p_{1Q}Q_1 + p_1 - C_{0q} - C_{1q} < 0$ . Therefore,  $\frac{\partial \Delta R}{\partial t_s} > 0$ .

Moreover, if a pair of taxes  $(\tilde{t}_s, \tilde{t}_v)$  induces Ramsey fiscal revenues, i.e.  $R(\tilde{t}_s, \tilde{t}_v) = R^*$ , thus

$$\begin{aligned} \Delta R(\tilde{t}_s, \tilde{t}_v) &= [p_0q_0 + p_1Q_1 - C_0 - nC_1] - [p_0q_0 - C_0 + (\tilde{t}_v p_1 + \tilde{t}_s) Q_1] \\ &= [(1 - \tilde{t}_v) p_1 - \tilde{t}_s] Q_1 - nC_1 \\ &= \Pi_1. \end{aligned}$$

Therefore,  $\Delta R$  and  $\Pi_1$  have the same sign. If  $(\tilde{t}_s, \tilde{t}_v)$  is such that  $\Delta R(\tilde{t}_s, \tilde{t}_v) < 0$ , this means that  $\Pi_1(\tilde{t}_s, \tilde{t}_v) < 0$  and, as  $\Delta R$  is increasing with the specific tax, the specific tax  $t_s$  that induces  $\Delta R(t_s, \tilde{t}_v) = 0$  must be such that  $t_s > \tilde{t}_s$ . The converse holds when  $\Delta R(t_s, \tilde{t}_v) > 0$ .